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Streaming of fluid under a near-bottom membrane

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Experiments are described which show the steady streaming of fluid under a flexible membrane, located near the bottom of a wave basin in shallow water. Circulation of fluid in a closed circuit was thus obtained, and measurements were made of the pressure head and total flow. The phenomenon might be used to extract power from waves at very low efficiency, but the theoretical basis is not yet understood.

1. Introduction

The classical work of Stokes (1847) on gravity waves in water of uniform depth established that in a progressive, irrotational wave the orbital motion is generally accompanied by a mean horizontal drift, or streaming, of the individual particles. The action of viscosity in modifying the fluid streaming, particularly near the upper and lower boundaries of the fluid, has been investigated by Longuet-Higgins (1953), Unluata & Mei (1970) and others.

In view of the recent interest in extracting power from sea waves (see e.g. Count 1980), the present author performed a series of laboratory experiments to investigate whether it might be possible to harness the phenomenon of fluid streaming for practical purposes. The proposed device (Allison 1979) is shown diagrammatically in figure 1. This consisted of an array of tubular elements located on a seabed in shallow water, each roofed by a flexible membrane filled with a fluid, the ends of tubular elements being connected by the conduits, which in turn are linked by pipes with a turbine installed on a shore. The underlying idea was that in shallow water the waves, propagating on the surface, would cause running waves of deflection in the flexible membranes, and that fluid under the membranes in the tubular elements would be streamed by these propagating waves of flexure in the membranes.

Such a device would have certain practical advantages over other methods of wave-power extraction, in that the moving parts would be few, and that the device would be protected from the violent action of breaking waves at the free surface.

However, no information on streaming of fluid beneath a flexible membrane seems to be available in the literature, and, indeed, it is not self-evident that the fluid will be streamed, because a counterstreaming may well be developed under the membrane, so that the net mass flux might well be zero at the end of tubular elements in figure 1. To clarify the matter, some experiments in a wave tank were performed, as described below. The chief aim of the experiments was to establish whether the fluid streaming actually occurs under the membrane, and, secondly, to determine the power available.

The experiment has shown that the phenomenon of fluid streaming under a near-bottom membrane exists, and the steady-state regime is achieved quite rapidly. The performance, in terms of fluid velocity and pressure heads, depends on the wave

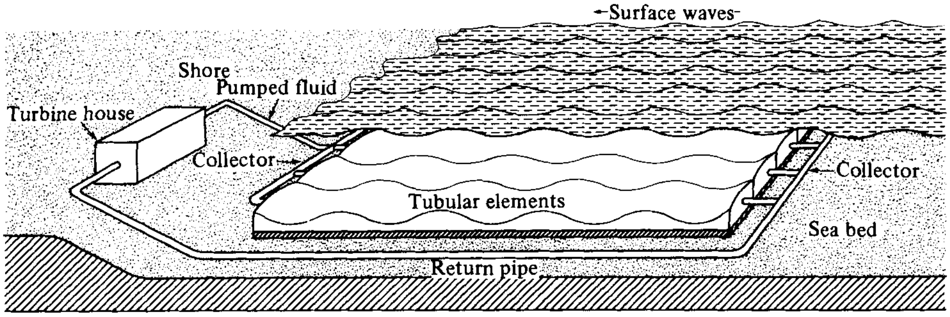

FIGURE 1. A schematic diagram of CSIRO (Australia) wave-power machine. An array of tubular elements is located on a seabed in shallow water. Propagating surface waves cause deflection of membranes at the top of the tubular elements, the deflections also propagating and thus driving a fluid under the membranes.

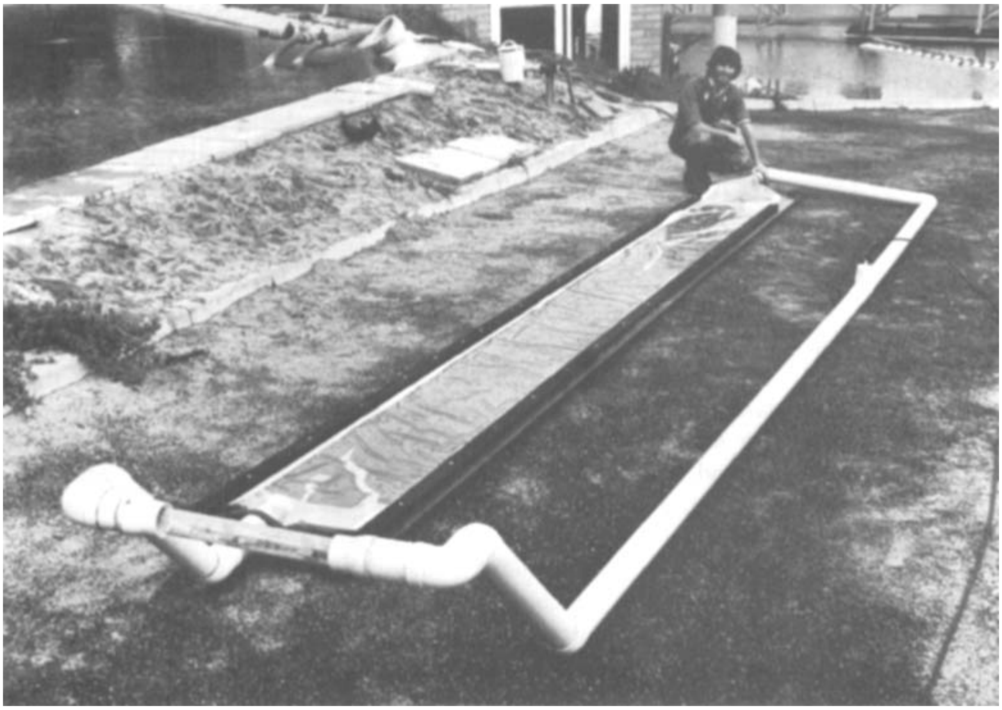

FIGURE 2. A single tubular element (before filling with fluid) with a return pipe and a propeller section, simulating the turbines. Plastic membrane on the top is seen collapsed.

period. Although the efficiency of the device may be low, we feel that the phenomenon itself is of some fluid-dynamical interest. A theoretical discussion is given in the companion paper by Longuet-Higgins (1983).

2. Experimental layout

The model of just one tubular element of the array shown in figure 1 was tested in the wave tank (figures 2 and 3). A portion of the return pipe was raised above the water level in the wave basin, and a small propeller was installed in this transparent

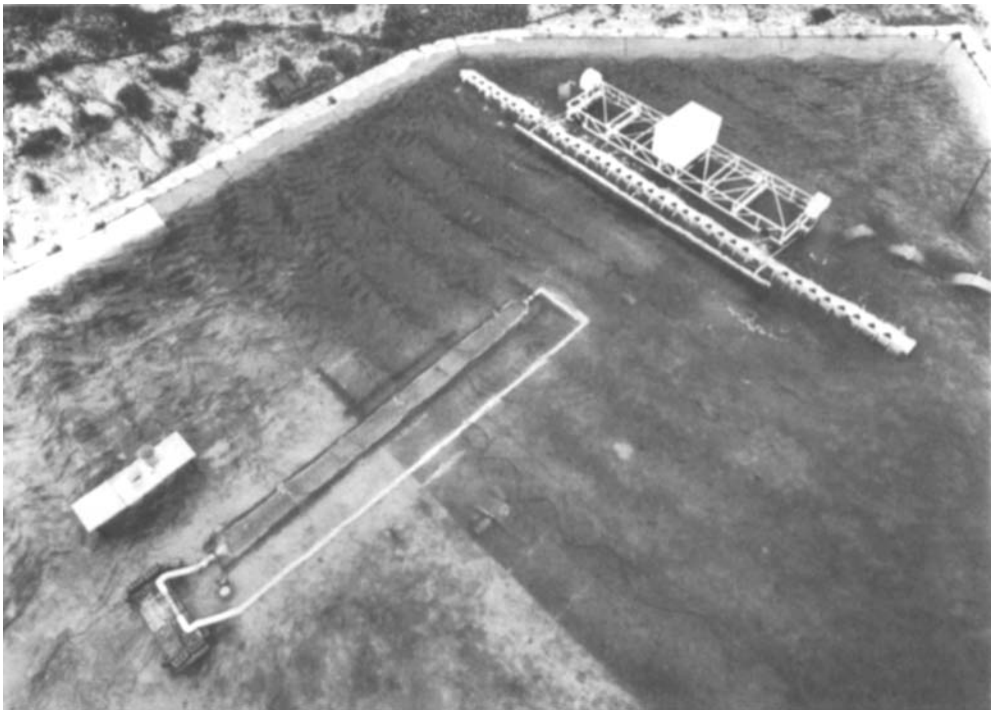

FIGURE 3. The element under test in a wave basin. Flow velocity in return pipe is being recorded.

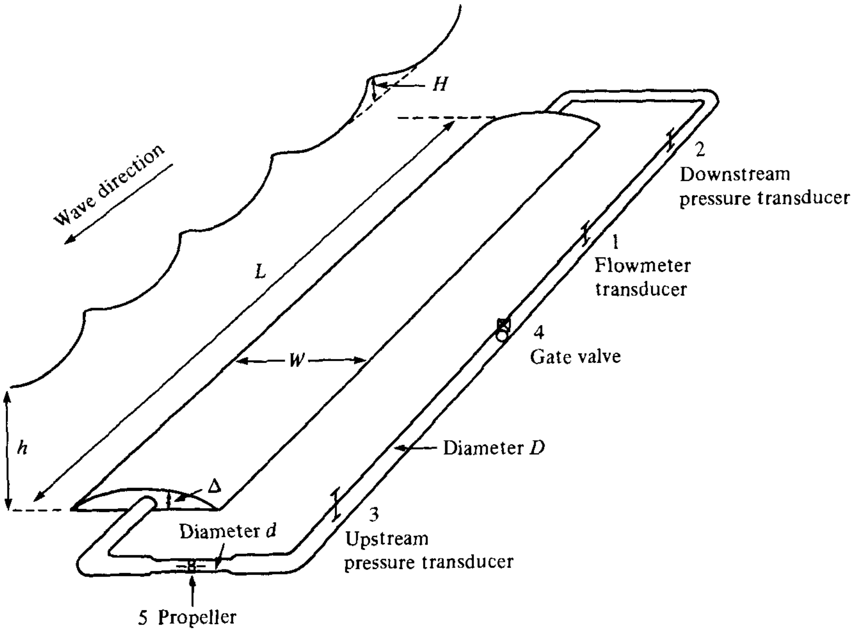

FIGURE 4. Schematic diagram of a tubular element; $\Delta \approx 4$ cm, $W = 50$ cm, $D = 9$ cm, $d = 5.4$ cm, $h = 30$ cm, $L = 600$ cm, $H \approx 5$ cm. 1, electromagnetic flowmeter tranducer; 2, downstream pressure transducer; 3, upstream pressure transducer; 4, gate value; 5, propeller.

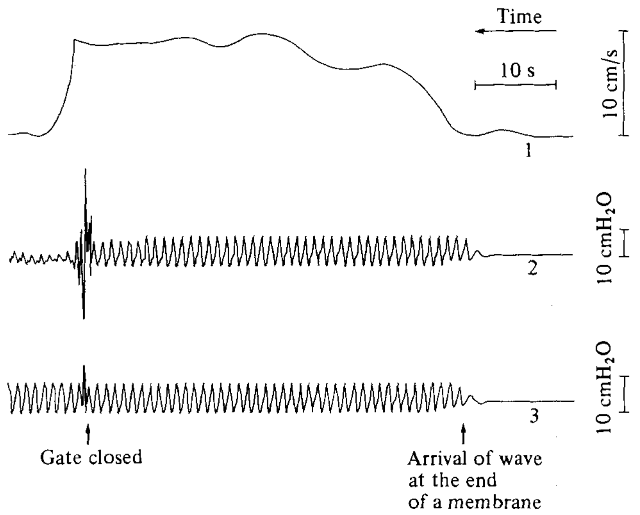

FIGURE 5. Transient processes due to arrival of a first wave (A) and closing the gate-valve (B). Trace 1 is a flowmeter record, filtered with 1 s time-constant filter. Traces 2 and 3 are unfiltered records of pressure. Labelling of traces corresponds to labelling of the instruments in figure 2.

section of the pipe, simulating the water turbines in figure 1. After switching on the wavemaker (figure 3), it only required waves to reach the rear end of the tubular element, for the propeller to start to rotate, thus indicating the presence of a flow through the tubular element and the return pipe.

For more elaborate experiments, instruments for measuring flow velocity and pressure were installed in the return pipe, together with a gate valve, imitating the external load (hydraulic resistance); see figure 4. Electric signals from the pressure transducers (type WHM-2 by Hales & Rogers Pty, Australia) and a flow-velocity transducer (type Marsh–McBirney, model 511, USA) were recorded on a strip-chart recorder. The signals had the option of being filtered electronically with time constants from 0.2–100 s. A short filtering time was used to obtain the instantaneous values of pressure variation and flow velocity. For recording mean values, averaged over a number of wave periods, time constants of the order of 10–100 s were used.

Sudden closure and opening of the valve provided the means of recording the transient regimes, analogous to short-circuit and open-circuit tests in electrical generators. Various degrees of opening of the gate valve were tried in order to simulate a variable loading. Finally, variations in wave periods on flow velocity and pressure were studied.

3. Experimental results

3.1. *Transient regimes*

The transient process of establishing a steady flow with the gate valve fully open and the wavemaker switched on is shown in figure 5. The time of arrival of the first wave towards the rear end of the membrane is marked by an arrow. After only 5–6 wave periods a steady-state flow regime is achieved. The flow-velocity record in figure 5 shows a steady streaming, while the pressure records demonstrate oscillations, strongly asymmetric in shape, thus pointing to nonlinearity of the process.

Sudden closure of a gate valve is depicted in figure 6, where the flow is seen to

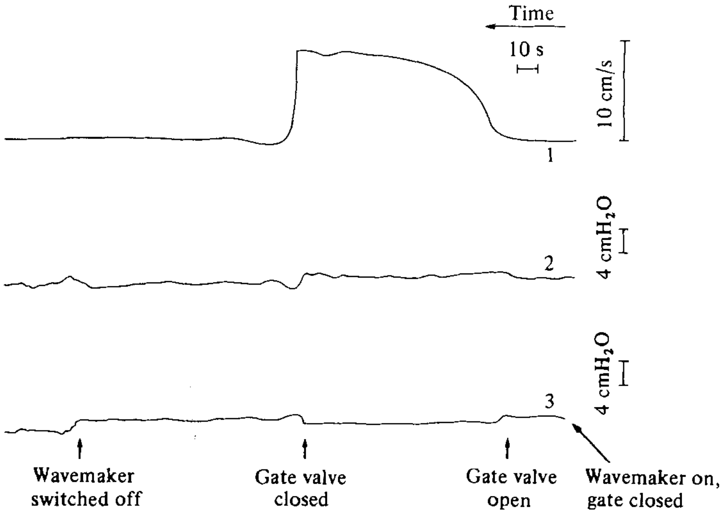

FIGURE 6. Transient processes due to opening/closing the gate valve and switching off the wavemaker. Trace 1: the flowmeter with 1 s time constant. Traces 2 and 3: pressure-transducer records with time constant of filtering 10 s, thus providing the mean pressure.

Gate-valve opening (in % of cross-sectional area)	Fully open	30 % open	12 % open	3 % open	Closed
Mean driving pressure over a gate valve (cmH$_2$O)	0	0.75	0.95	1.3	1.6
Flow velocity (cm/s)	10.3	6.0	4.0	1.0	0

TABLE 1

diminish rapidly and a negative pressure (suction) develops at one side of the gate valve, while the positive pressure increases at the other side.

In figure 6 the response to a sudden opening and closing of the gate valve is recorded with a 10 s filter time constant; hence the pressure can be regarded as the mean pressure. Longer time constants of filtering (up to 100 s) did not show much change in the mean pressure values as compared with the record in figure 6, except that filtering with longer time-constants tended to smooth the transient process record.

3.2. *Steady-state regime with a variable hydraulic load and internal static pressure*

Values of mean pressure over a gate valve and flow velocity, averaged over a number of experimental runs, are given in table 1 as a function of the relative opening of the gate valve. These results correspond to wave heights of about 5 cm in the basin, wave periods of the order of 1.1 s and the static pressure inside the tubular element exceeding the hydrostatic pressure in the wave tank by about 2 cmH$_2$O. This latter factor was found to be of significance for performance of the membrane.

This is illustrated in figures 7 (*a*, *b*), where discharge through the return pipe and driving pressure, measured as the mean (filtered) pressure difference at the ends of a

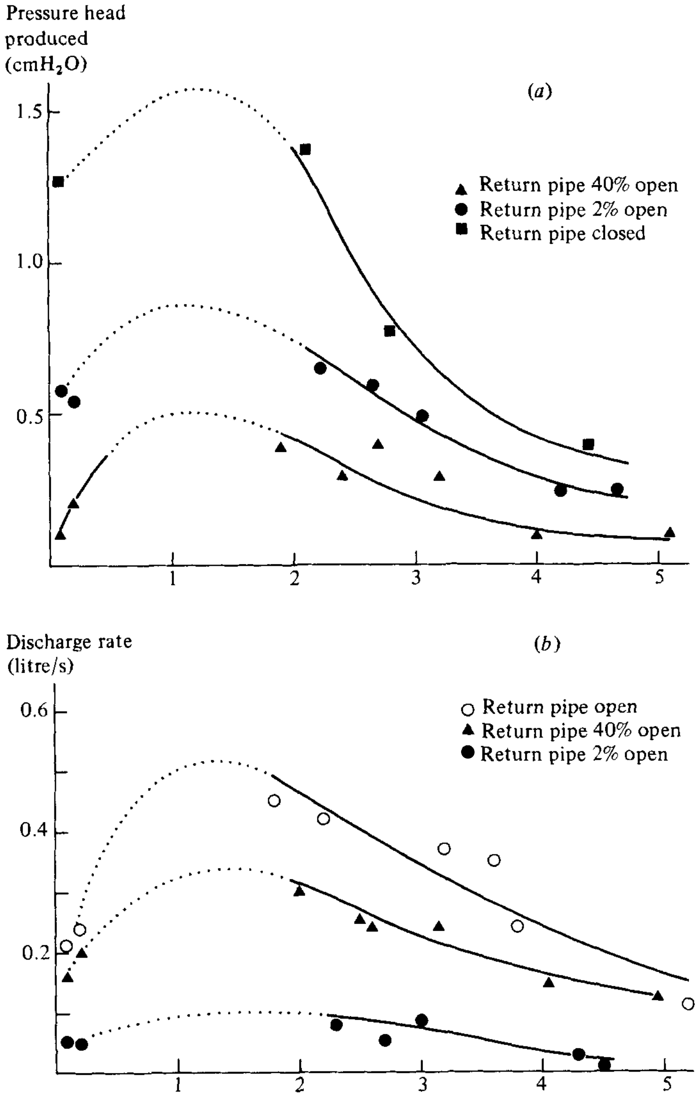

FIGURE 7. Variation of mean pressure head (*a*) and discharge rate (*b*) due to excess of internal pressure over the hydrostatic pressure.

tubular element, are plotted versus an internal hydrostatic pressure. One explanation, proposed by Longuet-Higgins (personal communication), is that a certain optimal pressure is needed to keep the gap between a membrane and the bottom of a tubular element (Δ in figure 4) at the optimal size. The size of the gap should not permit the interaction of the upper and lower boundary layers inside the tubular element.

Another possible explanation involves the necessity to maintain a certain optimal stiffness of the membrane as well as the optimal gap.

3.3. *Effect of a variable wave period*

Internal peak-to-peak pressure in the return pipe and the flow velocity in it are plotted versus wave period T in figure 8. Wave heights in the wave tank varied with

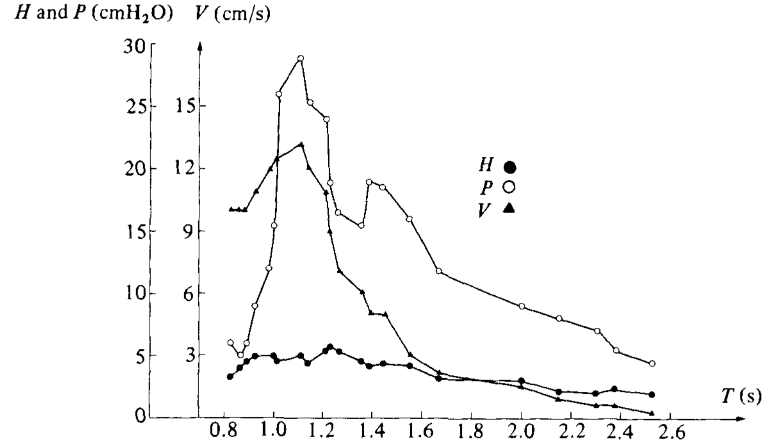

FIGURE 8. Effect of a variable wave period T on peak-to-peak pressure P (open circles) and flow velocity V (triangles). Variation of wave heights H in the wave tank (solid dots) with period T are also plotted. Curves joining the dots are drawn only for ease of following the sequences of dots.

the period. Nevertheless, the sharp peak of flow velocity and pressure near the period of 1.1 s occurred in the region of fairly stable wave heights (about 5 cm) in the wave tank.

At this stage we can only speculate that, possibly, the resonance is caused by the formation of partially standing waves of the membrane deflection, owing to their partial reflections from the ends of a tubular element; the mass of fluid under the membrane and the stiffness of the membrane determining the resonance period. If this is so, control of the internal static pressure in the tubular elements of figure 1 can provide a convenient way of tuning the tubular elements for predominant wave periods in actual sea conditions.

4. Discussion

From the data obtained in our experiments one may estimate the mechanical efficiency of the proposed device on a laboratory scale. The flux F of wave energy along a rectangular strip of width W is given by

$$F = Ec_g W,$$

where E is the wave-energy density and c_g the group velocity. On linear theory

$$E = \tfrac{1}{8}\rho g H^2$$

and

$$c_g = \frac{\sigma}{2k}\left(1+\frac{2kh}{\sinh 2kh}\right),$$

where k is the wavenumber, related to the frequency $\sigma = 2\pi/T$ by

$$\sigma^2 h/g = kh \tanh kh.$$

Taking as typical values from figures 8 and 2

$$T = 1.1\ \text{s}, \quad H = 5.0\ \text{cm}, \quad h = 30\ \text{cm},$$

we obtain

$$F = 2.0 \times 10^7\ \text{c.g.s. units}.$$

On the other hand, the power available for driving the turbine is certainly not greater than

$$Ag\,(VP)_{\max},$$

where A is the cross-sectional area of the pipe (i.e. 64 cm^2) and $(VP)_{\max}$ is the maximum value of the product of entries in lines 1 and 2 of table 1, that is 4.0 cm^2/s. Hence the power output cannot exceed about 2.5×10^5 c.g.s. units, and we may conclude that the mechanical efficiency is of order 1 %. This is low even compared with other wave-power devices (Count 1980), where mechanical efficiencies of 50 % or more have been achieved.

How the efficiency of the device varies with its scale must await a rational theory of the streaming mechanism. A theoretical discussion which explains some at least of the experimental results is given in the companion paper by Longuet-Higgins (1982). According to his interpretation, the streaming is driven mainly by viscous action, so that an increase in scale would tend to reduce the efficiency still further. On the other hand, the efficiency might be improved somewhat by the use of an internal working fluid having a higher viscosity.

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